E 7484 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2017

Third Semester

Core Course (3)—CALCULUS

(Common for Model I, Model II Mathematics and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. State Leibnitz's theorem.
2. Write the Maclaurin's series expansion of $\sin x$.
3. Define curvature of a curve at a point.
4. If $z = y^x$, find $\frac{\partial z}{\partial x}$.
5. State Euler's theorem on homogeneous functions.
6. Find $\frac{du}{df}$ where $u = x^2y + y^3$, $x = \log t$, $y = e^t$.
7. Write the formulas for the length of a smooth curve $y = f(x)$ from a to b.
8. State Pappus's theorem for volumes.
9. Define the double integral of a function of two variables over a bounded region in the co-ordinate plane.
10. Write the Jacobian of the co-ordinate transformation $x = g(u, v)$, $y = h(u, v)$.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. If $y = e^{-x}(ax + b)$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.
12. Find the radius of curvature at the point θ on the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

Turn over

13. Expand by Maclaurin's theorem $\log\left(1+e^x\right)$.

14. If $u=\log\left(e^x+e^y\right)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=1$.

15. If $z=\log\sqrt{x^2+y^2}$, prove that $\frac{\partial^2 z}{\partial x}+\frac{\partial^2 z}{\partial y^2}=0$.

16. Find the asymptotes of $x^3+3x^2y+3xy^2+2y^3=x^2+y^2+x$.

17. Find the area of the region enclosed by $x = 2y^2$, $x = 0$ and $y = 3$.

18. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

19. Find the length of the curve $y=\frac{4\sqrt{2}}{3}x^{3/2}-1, 0\le x\le 1$.

20. For the integral $\int_0^2\int_{x^2}^{2x}(4x+2)\,dy\,dx$. Write an equivalent integral with the order of integration reversed.

21. Evaluate $\int_0^{\pi}\int_0^{x} x\sin y\,dy\,dx$.

22. Evaluate $\int_0^1\int_0^1\int_0^1\left(x^2+y^2+z^2\right)dz\,dy\,dx$.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. If $y=\cos\left(m\sin^{-1}x\right)$, prove that $\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}+\left(m^2-n^2\right)y_n=0$.

24. Expand $\sin^{-1}x$ in powers of x by Taylor's theorem.

25. Find the evolute of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

26. Find the maxima and minima of $f(x,y)=x^3+y^3-3x-12y+20=0$.

27. Verify Euler's theorem for $u=\sqrt{x^2-y^2}\,\sin^{-1}(y/x)$.

28. Find the area of the region in the first quadrant that is bounded above by $y=\sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

29. A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crossed the first plane at a 45° angle at the centre of the cylinder. Find the volume of the wedge.

30. Evaluate $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2}(\rho\cos\phi)\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$.

31. Solve the system $u = x - y$, $v = 2x + y$ for x and y in terms of u and v. Then find the value of the Jacobian $\partial(x, y)/\partial(u, v)$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) If $y = \sin\log\left(x^2 + x + 1\right)$, prove that $\left(1+x^2\right)y_{n+2} + (2n+1)(x+1)y_{n+1} + \left(n^2+4\right)y_n = 0$.

(b) Find the envelope of circles which pass through the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and their centres are upon its circumference.

33. Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $z = x + y$.

34. (a) Find the area of the surface generated by revolving $x = y^3/3$, $0 \le y \le 1$ about y-axis.

(b) Evaluate $\iint_R e^{x^2+y^2}\, dy\, dx$ where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

35. Find the centroid (J = 1) of the solid enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the paraboloid $z = x^2 + y^2$ and below by the xy-plane.

(2 × 15 = 30)